

How to Optimize the Decision Making Using Ensemble Probabilistic Forecasts

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Abstract

In this study, we take the 0-6 h probabilistic quantitative precipitation forecasts (PQPFs) as an example to illustrate how to evaluate the economic value (EV) of ensemble probabilistic forecasts (EPFs) and offer examples for users to understand how to use the EPFs to optimize their decision-making. The PQPFs are generated from the ensemble prediction system (EPS) of the Local Analysis and Prediction System (LAPS) operated at the Central weather bureau (CWB) in Taiwan.

In the EV analysis, it is assumed that users' cost and loss are explicitly known. Unfortunately, information of users' cost or loss is sometimes implicitly known. For example, farmers may wonder whether they should harvest their crops before the coming of a typhoon accompanied by heavy rainfall. Compared with normal harvesting, the action of harvesting in advance does not seem to need any cost; however, it may lead to a hidden loss since unripe crops are sold at a lower price. Furthermore, the farmers may also wonder what percentage (F , $0 < F \leq 1$) of crops should be harvested in order to minimize losses if they harvest crops in advance. This example implies that the cost and loss of farmers can be derived based on the experience of preventive actions through the EV analysis. Although the farmers' expected expense based on the LAPS PQPFs will vary with the F , the maximum EV (EV_{\max}) provided by the LAPS remains the same regardless of the F . In addition, the "full harvest" action yields minimal long-term average losses.

Keywords: probabilistic quantitative precipitation forecasts (PQPFs), economic value (EV), ensemble prediction system (EPS), decision making

1. Introduction

Different from the deterministic forecasts (DFs), the ensemble probabilistic forecasts (EPFs) consider uncertainties during the forecast process (e.g., initial errors, nonlinear dynamic errors, and model errors), and convey the uncertainty information to the users using probability. For a skillful EPS, the divergence of ensemble forecasts causes small forecast probabilities (P_f), which indicate that the possibility of the occurrence of a specific weather event is low and that the forecast uncertainty is large (i.e., the event cannot be predicted correctly). However, compared with the DFs with indication of "Yes" or "No" only, can such probabilities or uncertainties information really benefit the users or confuse them in decision-making?

Users who were accustomed to DFs felt confused if they were unfamiliar with the EPFs. For example, for farmers who care about whether air temperature is below 0°C (i.e., frost or chilling injury), the information provided by the DFs is a deterministic answer that "the

temperature tomorrow is 5°C ". This information does not indicate the reliability of the forecasts, but the farmers assume that the forecast is completely correct and make their decisions based on the forecast. However, the information provided by the EPFs is "the chance that the temperature tomorrow below 0°C is 70%". Farmers who are not familiar with the meaning of EPFs might have difficulties to make a decision, because they are not sure whether a probability of 70% indicates that the event will happen or that it will not happen. Therefore, users most frequently wonder whether an optimal probability threshold (P_t) can be provided along with the EPFs, thereby allowing them to take preventive actions (or make decisions), such as closing roads, harvesting crops in advance, and suspending work and school, when the P_f exceed the optimal P_t .

Regarding the decision-making, users wonder how to best use the EPFs for decision making to minimize their expected expense. Considering two farmers who grow different crops in the same geographical area, their crops are assumed to be affected when the rainfall is more than 20 mm (6 h)⁻¹; therefore, they have to take disaster

prevention. The protectable loss, which is defined as the avoidable loss after taking the prevention, for these two farmers could be very different depending on the values of the crops and the tolerance to different natural damages. If these two farmers pay the same cost of protection but their protectable loss is very different, the criteria used by the two farmers to take preventive action should be different (i.e., having different optimal P_t), and the farmer with larger protectable loss is prone to take action. After all, how do these two farmers use the EPFs to lower the cost of prevention or decrease the losses of crops? In Zhu et al. (2002), an example for using the economic value (EV) analysis was demonstrated by explicitly knowing the user's cost and loss. Unfortunately information of users' cost or loss (and thus the cost-loss ratio) is sometimes implicitly known. For example, farmers will wonder whether they should harvest their crops in advance before the coming of a typhoon accompanied by heavy rainfall. Compared with normal harvesting, the action of harvesting in advance does not seem to need any cost; however it may lead to a hidden loss since the unripe crops are sold at lower price. In this situation, is it possible for the farmers to optimize their decision making by using the EPFs?

In this study (Chang et al. 2014), we provide two examples to illustrate how to optimize the decision-making via the EV analysis, one with and the other without an explicitly known cost-loss ratio. This report is organized as follows: LAPS EPS and data are introduced in section 2. Section 3 describes the methodology for computing EV. Section 4 presents the application of LAPS PQPFs. A summary and future works are given in the last section.

2. LAPS ensemble prediction system and data

The 0-6 h probabilistic quantitative precipitation forecasts (PQPFs) used in this study are generated from ensemble forecasts based on the Local Analysis and Prediction System (LAPS). We adopted the time-lagged multimodel ensemble configuration to construct the LAPS ensemble prediction system (EPS), which has 12 members and can provide operational 0-6 h PQPF every three hours.

Chang et al. (2012) showed that the LAPS EPS has a good spread-skill relationship and skillful discrimination ability, and thus can be regarded as an EPS with good quality and predictive capability. The data used in this study (same as in Chang et al. 2012) for evaluating the EV include a total of 148 cases of 0-6 h PQPFs based on all typhoon cases in 2008 and 2009.

A calibration method based on linear regression has been used to calibrate the wet-biased PQPFs. Chang et al. (2012) show that this calibration method successfully corrects the wet bias.

3. Economic Value (EV)

a. Methodology

The EV of a forecast system (Richardson 2000) is defined as:

$$EV = \frac{E_{climate} - E_{forecast}}{E_{climate} - E_{perfect}} \quad (1)$$

where $E_{climate}$, $E_{forecast}$ and $E_{perfect}$ are the expected expenses of a user who takes preventive action based on the climatological information, a forecast system, and a perfect deterministic forecast system, respectively. A perfect forecast system means that it always provides accurate predictions for the occurrence and non-occurrence of a particular event. Therefore, $E_{perfect}$ is the smallest among these three expected expenses. According to the above definition, the EV can be interpreted as the relative performance taking the climatological information as a baseline. For example, if a perfect forecast can save the user 100 dollars, then a forecast system with economic value EV will save the user $100 \times EV$ dollars.

Values of EV range from minus infinity to 1. The maximum value $EV=1$ is obtained from a perfect forecast system, and $EV=0$ if climatological information is adopted ($E_{forecast} = E_{climate}$). Only when the $E_{forecast}$ is less than $E_{climate}$, it is beneficial for users to abandon the climatological information and make decisions based on the forecast system information.

In the EV analysis, we assume that a user takes action depending on the forecast information (i.e., this user takes action only when the event is predicted). Therefore, based on the past long-term performance of forecasts, we can evaluate the EV of a forecast system using a 2×2 contingency table (Table 1). Table 1 lists the relative frequencies and expected expense of a user for four outcomes, where C is the cost of preventive action, L ($L = L_p + L_u$) is the total loss caused by weather events, including the protectable loss (L_p) and unprotectable loss (L_u) after taking preventive action. If the preventive action can avoid the total loss, $L_u = 0$. The cost-loss ratio (expressed as $r = C/L_p$) is unique for each user since the corresponding C and L_p are different. Given that users take preventive action only when $L_p > C$, the value of r is between 0 and 1. In addition, the users with a very small r value are expected to have the large economic benefits, because they only need to pay marginal cost (C) to avoid a considerable protectable loss (L_p).

Using the definition in (1), Zhu et al. (2002) showed that EV can be expressed as

$$EV = \frac{\min[\bar{o}, r] - (h + f)r - m}{\min[\bar{o}, r] - \bar{o}r} \quad (2)$$

This equation shows that EV is related not only to the forecast performance (i.e., forecast parameters h , f , and m) but also to the climatological frequency (\bar{o}) of a particular weather event and the cost-loss ratio (r) of a user.

b. EV Analysis of the LAPS EPS

With 12 members, the LAPS EPS in this study provides 12 P_t values (i.e., 1/12 to 12/12). When P_f reaches (bellows) the P_t threshold, the weather event is declared as “will occur” (“will not occur”). Fig. 1a shows a set of EV curves at the 10 mm (6 h)⁻¹ threshold using 12 P_t generated from the LAPS 0–6 h calibrated PQPFs. The choice of P_t by a user has a decisive influence on the EV that she/he could obtain. For example, a decision maker with $r = 0.1$ will obtain an EV of 36% based on $P_t = 2/12$ (Fig. 1b), but will only obtain an EV of 16% when a higher probability threshold ($P_t = 4/12$) is adopted, and this decision-maker no longer gains any EV by using the LAPS PQPFs with a P_t larger than 5/12. In comparison, for a decision maker with a larger r of 0.25 (=3/12), who supposes to obtain an EV of 51% based on $P_t = 3/12$, can only obtain 11% EV when adopting the criterion $P_t = 8/12$. Therefore, the EVs for different decision-makers and the optimal P_t required to maximize their EV are user-dependent. Murphy (1977) showed that if perfectly reliable (i.e., unbiased) forecasts are adopted, the optimal P_t for maximizing EV is equal to the r value of users. Different users should choose the optimal P_t based on their r so that their EV can be maximized.

The maximum EV (EV_{\max}) that can be provided by the LAPS EPS for users is shown as the envelope of all EV curves (Fig. 1c). The area under the EV_{\max} curve is called the potential EV. At the 10mm (6 h)⁻¹ precipitation threshold, users with r between 0.025 and 0.8 have positive EV values and thus can benefit from making decisions by referencing the LAPS PQPFs, indicating they may incur smaller expected expenses than using climatological information. As mentioned in Richardson (2000), the highest value of EV_{\max} can be obtained by the users whose r value equals to climatological frequency \bar{o} [e.g., $\bar{o} = 28\%$ for the 10 mm (6 h)⁻¹ threshold].

4. Application of LAPS PQPFs

This section offers examples of EV applications for users to understand how to use the EPFs in daily life to optimize their decision-making. Note that the following examples require the assumption that calibrated LAPS PQPFs (i.e., near perfectly reliable) are used. If uncalibrated PQPFs are used, users must employ the EV distributions from past long-term statistical samples to determine the optimal P_t .

a. An example with explicitly known cost-loss ratio

We review the example mentioned in the introduction. Two farmers are growing different crops in the same area. When the precipitation intensity exceeds 20 mm (6 h)⁻¹, both farmers must take a preventive action, such as building rainproof structures; otherwise, their crops will be damaged. Assume that the cost-loss ratio is 6/12 for Farmer 1 and it is 1/12 for Farmer 2. If a missing rate of 11% and hit rate of 41% is given by the

deterministic forecast from LAPS-WRF (NFS) model for precipitation intensity larger than 20 mm (6 h)⁻¹ that may cause losses in growing crops, only users with r between 0.11 and 0.41 obtain economic value (Fig. 2a). Such a deterministic forecast can't offer any economic value to these two farmers; therefore, they would rather make decision based on the climatological information instead.

The climatological information indicates the frequency of this rainfall event is 17%. Based on this information, Farmer 1 ($r > \bar{o}$) must choose “never” taking preventive action, while Farmer 2 ($r < \bar{o}$) must choose “always” taking preventive action to minimize their expected expense from the statistical point of view in terms of long term.

Instead of a single P_t ($P_t = 100\%$) like deterministic forecasts, the LAPS PQPFs (Fig. 2b) can provide different P_t , ranging from 1/12 to 12/12. With P_t values of 1/12 and 6/12, both farmers can make decision to obtain their own EV_{\max} . With such kind of PQPFs, Farmer 1 no longer chooses “never taking prevention” based on the climatological information; rather, he or she should take action when $P_f \geq 6/12$ to decrease the crop losses. Similarly, Farmer 2 no longer chooses “always taking prevention”; rather, he or she needs to take action only when $P_f \geq 1/12$ to lower the cost of disaster prevention. Therefore, such kind of PQPF allows both farmers to obtain an economic value greater than that based on climatological information or the deterministic forecasts.

b. An example without explicitly known cost-loss ratio

During the typhoon seasons in Taiwan, farmers of Chinese dates are concerned about whether the typhoon will be accompanied by heavy rainfall, because it will result in date cracking and reduce the quality of the dates. If heavy rainfall [say, ≥ 20 mm (6 h)⁻¹] is likely to occur, the farmers must decide whether they should harvest dates in advance to minimize their losses. Two conditions are considered in relation to the price drop: a premature harvest and being affected by heavy rainfall. The ratios between the reduced and original prices for these two conditions are denoted as R_1 and R_2 , respectively. Both ratios are stationary and can be obtained based on the experience of preventive actions. Note that in this example, some factors are not considered yet, such as that the wages of labor for harvesting fruits might become more and more expensive as the lead time is getting shorter and shorter. Assume two weeks before dates ripen, the LAPS PQPF indicates a 50% probability of rainfall ≥ 20 mm (6 h)⁻¹ in the coming six hours, should the farmers harvest dates before the advent of the heavy rainfall?

Assuming that the total price of ripe dates with normal harvest is A , R_1 is 80%, and R_2 is 40%, we can use Table 1a to analyze the expected expense of farmers in four possible situations. In situation 1, harvesting in advance and heavy rainfall not occurring (the expected expense of the farmers is C); in this situation, compared with normal harvesting, the action of harvesting in

advance does not seem to pay any cost; however, it does reduce the total income of the farmers. This reduction in income should be regarded as the cost of preventive action; therefore, the expected expense $C = (1-80\%)A$. Following the same concept, the expected expense is calculated by considering the reduction in the income of the farmers in the remaining three situations. In situation 2, harvesting in advance and heavy rainfall occurring, the expected expense $(C+L_u) = (1-80\%)A$. Note that the expected expense in situation 2 is the same as that in situation 1, which reveals that no need to consider L_u in this case. This is understandable since after harvesting all the dates in advance, the farmers will not incur any unprotectable loss, L_u , when the heavy rainfall occurs (i.e., $L_u = 0$). It should be noted that the farmers will incur some unprotectable loss if only harvesting part of the dates since the heavy rainfall will damage the unharvested dates. In situation 3, no preventative action and heavy rainfall occurring, the expected expense $(L_p+L_u)=L_p=(1-40\%)A$. In situation 4, no preventative action and heavy rainfall not occurring, the expected expense $N=0$. With this contingency table, we can calculate the cost-loss ratio of the farmers, $r = C/L_p \sim 0.33$; therefore, if $P_f=50\%$, P_f is greater than the optimal P_t and the farmers should harvest in advance to reduce their losses. If, however, the forecast probability of heavy rainfall is 10%, the P_f is smaller than the optimal P_t and the farmers do not need to harvest in advance to lower the prevention cost.

By contrast, if the typhoon may hit four weeks before dates ripen, and the price ratio of premature dates to ripe dates is 60% (R_1), should the farmers harvest in advance? We can calculate the r of the farmers in the same way (Table 6b), and $r = C/L_p \sim 0.67$. Therefore, the farmers do not need to harvest in advance when P_f is 50% or 10%, because P_f for both cases is smaller than the optimal P_t . However, the farmers should harvest in advance when P_f is greater than the optimal P_t , (e.g. $P_f=90\%$).

Furthermore, the date farmers may also wonder what percentage (F , $0 < F \leq 1$) of dates should be harvested in order to minimize losses if they harvest dates in advance on the basis of the forecast information. Let us re-evaluate the expected expense in the first three possible situations but take into account the harvest percentage (F).

Situation 1 :

$$C = A - [R_1 A F + A (1-F)] = (1 - R_1)FA \quad (3)$$

Situation 2 :

$$\begin{aligned} C + L_u &= A - [R_1 A F + R_2 A (1-F)] \\ &= [(1 - R_2) + (R_2 - R_1)F]A \end{aligned} \quad (4)$$

Situation 3 :

$$L = L_p + L_u = A - R_2 A = (1 - R_2)A \quad (5)$$

Using (3) to (5), the values of L_u , L_p , and r are obtained as follows:

$$L_u = (13) - (12) = (1 - R_2) (1 - F)A \quad (6)$$

$$L_p = L - L_u = (14) - (15) = (1 - R_2)FA \quad (7)$$

$$r = C / L_p = (12) / (16) = (1 - R_1) / (1 - R_2) \quad (8)$$

This example shows that the r of farmers does not change as the F varies. Equation (2) indicates that the EV is linked to three factors: the forecast performance (h , f , and m), the $\bar{\sigma}$ of a weather event, and the r of a user. When h , f , m , $\bar{\sigma}$, and r remain unchanged, the EV also remains the same although varying F yields different C , L_u , and L_p . In other words, the economic value provided by this forecast system remains the same regardless of farmers' F .

However, the expected expense of farmers depends on the harvest percentage (F) no matter they adopt the information of the LAPS PQPF or the perfect forecast (note that E_{climate} may remain unchanged). The following equations show the expected expense according to three different sources of forecast information:

$$\begin{aligned} E_{\text{forecast}} &= h (C + L_u) + f C + m (L_p + L_u) \\ &= (1 - R_2)(h + m)A - [h(R_1 - R_2) - f(1 - R_1)] AF \end{aligned} \quad (9)$$

$$\begin{aligned} E_{\text{climate}} &= \min [\bar{\sigma}(L_p + L_u), C + \bar{\sigma}L_u] \\ &= \min \{ \bar{\sigma}(1 - R_2)A, \bar{\sigma}(1 - R_2)A + [(1 - R_1) - \bar{\sigma}(1 - R_2)]FA \} \end{aligned} \quad (10)$$

$$\begin{aligned} E_{\text{perfect}} &= h (C + L_u) = \bar{\sigma}(C + L_u) \\ &= \bar{\sigma}[(1 - R_2) + (R_2 - R_1)F]A \end{aligned} \quad (11)$$

Equation (9) indicates that E_{forecast} is the linear function of F . Under the situation that the farmers can obtain the EV_{max} , the E_{forecast} will be the expected expense of farmers when the optimal P_t is adopted.

Figure 3 shows how the expected expenses (E_{climate} , E_{forecast} , and E_{perfect}) and EV_{max} varied with the F ($0 < F \leq 1$) for the cases of two and four weeks before dates ripen. As mentioned above, the EV_{max} does not change with the F . In addition, the E_{forecast} reaches to minimum at $F=1$ in both cases, indicating the full harvest leads to the largest economic benefit. This can be understood as following.

If the EV_{max} is greater than zero (i.e., $E_{\text{forecast}} < E_{\text{climate}}$), the r value of farmers must range between the missing rate and the hit rate derived from the long-term forecast performance on the basis of optimal P_t :

$$\frac{m}{1 - (h + f)} < \frac{1 - R_1}{1 - R_2} < \frac{h}{(h + f)} \quad (12)$$

From the right inequality of (12), we obtain $[h(R_1 - R_2) - f(1 - R_1)] > 0$. Therefore, E_{forecast} decreases linearly with increasing F [Eq. (9)] and the minimum E_{forecast} is obtained when $F = 1$ (full harvest). In other words, when information of the forecast system indicates that dates must be harvested in advance, the "full harvest" action yields minimal long-term average losses.

7. Summary and future works

In the analysis of EV, we assume that the cost-loss

ratios of users are explicitly known. However, in some cases, information of users' cost-loss ratio cannot be obtained explicitly. In this study, we provide an example to illustrate that even if without explicitly knowing the r , users can still optimize their decision-making via the analysis of EV.

The implication of the example is that the r of farmers can be derived based on the experience of preventive actions through the EV analysis. Although the farmers' expected expense based on the LAPS information will vary with the F , the EV_{\max} provided by the LAPS remains the same regardless of the F . In addition, the "full harvest" action yields minimal long-term average losses.

In this study, parameters for EV analysis may be over simplified. For example, the wages of labor for harvesting fruits might become more and more expensive as the lead time is getting shorter and shorter. Therefore, the farmer's cost-loss ratio should vary in time [i.e., $r = r(t)$].

In addition, we use the 0–6 h PQPF exceeding a given threshold as an index of weather event for EV analysis. Such kind of weather indices contain only one meteorological field, but other EPF products can provide indices which combine information from multiple meteorological fields. Particular industries may be influenced by various meteorological factors (e.g., temperature and humidity) simultaneously. For example, farmers regard temperature and precipitation as vital indicators in terms of their economic benefits, fishermen are concerned about wind speed and wave height, and the wine industry about sunshine hours, temperature, and humidity. In the future, the CWB plans to cooperate with different industries to understand the weather forecast information that is important to them. This would enable the CWB to design EPF products delivering weather indices that specifically address the needs of different industries, thereby enabling these industries to achieve greater EVs.

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TABLE 1. Contingency table for forecasts and observations of a binary event.

| | | Forecast / action | |
|-------------|-----|---|--|
| | | Yes | No |
| Observation | Yes | Hit (h) Mitigated loss ($C+L_r$) | Miss (m) Loss (L_r+L_n) |
| | No | False alarm (f) Cost (C) | Correct rejection (c) No cost (N) |

TABLE 2. Contingency tables in case of (a) two weeks ($R_1=80\%$) and (b) four weeks ($R_1=60\%$) before dates ripen. Assume that $R_2=40\%$ and the total price of ripe dates with normal harvest is A .

(a)

| | | Forecast / action | |
|-------------|-----|--|---|
| | | Yes | No |
| Observation | Yes | Hit (h) (2) $C+L_r=(1-80\%)A$ | Miss (m) (3) $L_r+L_n=(1-40\%)A$ |
| | No | False alarm (f) (1) $C=(1-80\%)A$ | Correct rejection (c) (4) $N=0$ |

(b)

| | | Forecast / action | |
|-------------|-----|--|---|
| | | Yes | No |
| Observation | Yes | Hit (h) (2) $C+L_r=(1-60\%)A$ | Miss (m) (3) $L_r+L_n=(1-40\%)A$ |
| | No | False alarm (f) (1) $C=(1-60\%)A$ | Correct rejection (c) (4) $N=0$ |

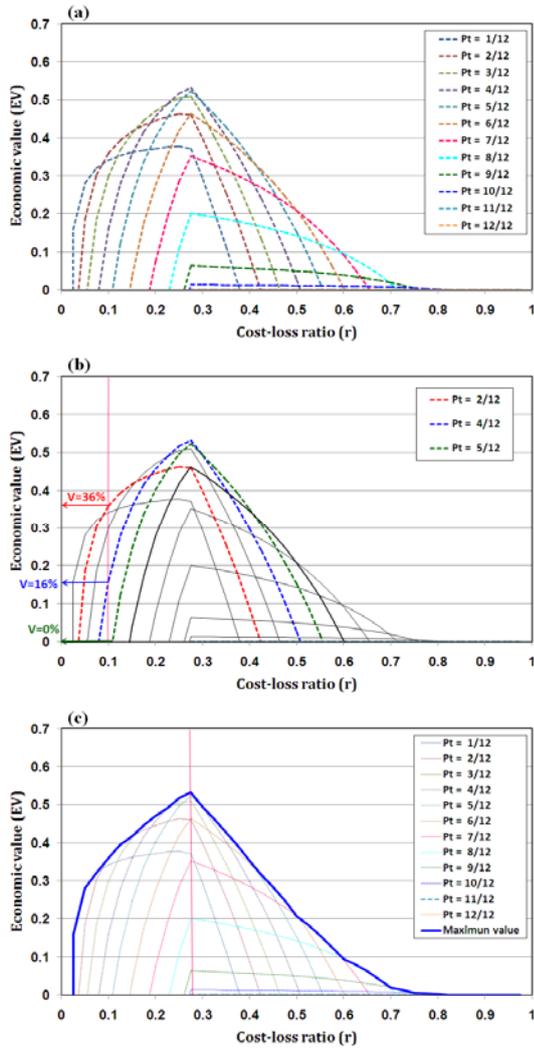


FIG. 1. For the LAPS 0-6-h PPDFs at the 10 mm $(6 \text{ h})^{-1}$ threshold, (a) economic value (EV) against the cost-loss ratio (r) at different probability thresholds ($P_t=1/12$ to $12/12$), (b) illustration of economic values obtained by users with $r = 0.1$ when adopting different P_t values ($P_t = 2/12, 4/12$ and $5/12$), (c) distribution of the maximum economic value (EV_{\max}) if adopting the optimal P_t . The straight red line indicates the greatest EV_{\max} obtained by users with $r = \bar{\delta}$.

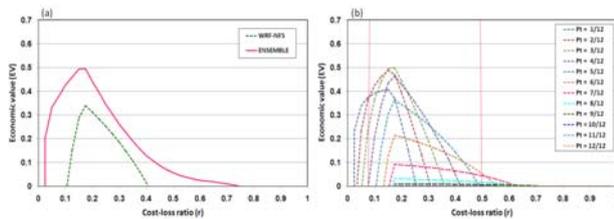


FIG. 2. At the threshold of 10 mm $(6 \text{ h})^{-1}$, (a) the distribution of economic value from the LAPS 0-6-h PPDFs (red curve) and the 0-6-h QPFs from the LAPS-WRF(NFS) model (green curve); (b) the economic value curves against cost-loss ratio (r) at different probability thresholds ($P_t=1/12$ to $12/12$) from LAPS calibrated 0-6-

h PPDFs. The straight red lines indicate the cost-loss ratios of Farmer 1 ($r=6/12$) and Farmer 2 ($r=1/12$).

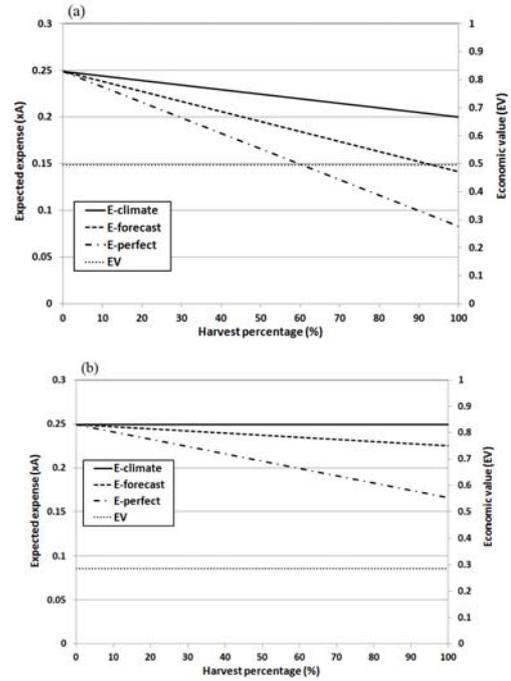


FIG. 3. The expected expense (E_{climate} , E_{forecast} , and E_{perfect}) and economic value (EV) as a function of harvest percentage (F , $0 < F \leq 1$) in case of (a) two weeks and (b) four weeks before dates ripen when the optimal P_t is adopted. A is the total price of ripe dates with normal harvest.