Stochastic Simulation and Frequency Analysis of the Concurrent Occurrences of Multi-site Extreme Rainfalls

Guan-Ming Su, Tsung-Hsun Lien, Ke-Sheng Cheng
Department of Bioenvironmental Systems Engineering
National Taiwan University

Abstract

Traditionally, hydrological frequency analyses were independently conducted at individual sites using annual maximum rainfalls. While this approach can provide useful results for site-specific design rainfall depths, it fails to offer insightful information about the return period of the concurrent occurrence of multi-site extreme rainfalls. There have had many evidences that a single major storm can result in annual maximum rainfalls of different durations at several sites, as demonstrated by the Typhoon Morakot in 2009. Thus, spatial correlation of the same-event rainfall depths at different sites plays an essential role in characterizing the concurrent occurrences of multi-site extreme rainfalls. In this study, we propose an innovative approach of multi-site storm rainfalls simulation to tackle the problem of frequency analysis of multi-site extreme rainfalls. The approach is composed of four major components: (1) delineating homogeneous regions within the study area using K-means cluster analysis, (2) standardizing and modeling the same-event rainfall depths at different sites by a Pearson Type III random field, (3) stochastic simulation of multi-site storm rainfalls using a covariance matrix conversion algorithm, and (4) determining the return periods of concurrent occurrences of multi-site extreme rainfalls. Using historical hourly rainfall records available at 25 rainfall stations in southern Taiwan, site-specific rainfall depths of different durations and return periods of this study were found to be very close to results of a previous study of single-site frequency analysis, indicating good applicability of the proposed approach. Additionally, the return period of four specific sites within the Kaoping River Basin (Shan-di-men, Ah-li, Jia-sien and Chi-shan) exceeding 1000 mm rainfalls in a 24-hour period, a scenario similar to rainfalls of Typhoon Morakot, was found to be only 514 years, as oppose to more than 2000 years or even higher by other previous studies.

Key word: Stochastic simulation, multi-site extreme rainfalls, frequency analysis, variogram

1. Introduction

Rainfall extremes of certain design durations and return periods are essential elements in hydrological design. Traditionally, such rainfall extremes are required at individual rainfall stations and are determined by single-site frequency analysis. However, there have had many evidences that a major storm (such as Typhoon Morakot of 2009) can result in extreme rainfalls at several neighboring stations and the return period of such concurrent occurrences of multi-site rainfall extremes cannot be determined by single-site frequency analysis. A unique feature of the concurrent occurrences of rainfall extremes is the significant spatial correlations among rainfalls of different stations. Taiwan has experienced increasing intensity, longer duration, and more extensive rainfall extremes of typhoons in recent years. Thus, it is due necessary for hydrological frequency analysis considering concurrent occurrences of multi-site rainfall extremes. In this study, we propose a new stochastic approach for simulation of event-specific multi-site maximum rainfalls with respect to certain design durations. The proposed approach is capable of generating large number of realizations of multi-site typhoon rainfalls which can then be used for multi-site frequency analysis of rainfall extremes.

2. Study area and data

Forty seven years (1965 – 2011) of hourly rainfall data available at twenty five rainfall stations in southern Taiwan (see Fig. 1) were collected. Since almost all long-duration (≥ 12 hours) rainfall extremes were produced by typhoons, we firstly extracted hourly rainfall series of individual typhoon events at different rainfall stations. From these data, event-maximum rainfalls of 8 design durations (1, 2, 6, 12, 18, 24, 48 and 72 hours) of individual typhoons at all rainfall stations were obtained. Event-maximum rainfalls at individual stations were then standardized with respect to their long term averages and standard deviations. These duration-specific standardized event-max rainfalls have zero expectation and unit variance and their spatial variations were modeled as a Pearson type III random field.

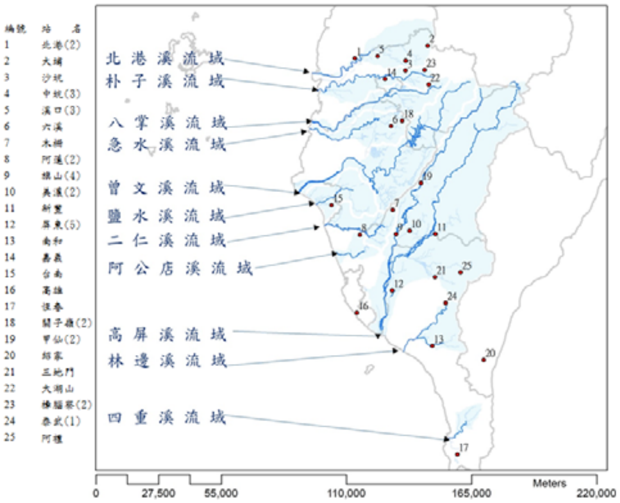

Figure 1. Study area and locations of rainfall stations.

3. Delineation of homogeneous regions

Event-maximum rainfalls within the study area vary with locations due to orographic effect. It is a common practice to delineate a few homogeneous regions which can reflect the orographic effect within the study area. The K-means cluster analysis was conducted by using the mean, standard deviation and coefficient of skewness of the event-maximum rainfalls of the eight durations as feature variables. The 25 rainfall stations were classified into two homogeneous regions with stations 1 – 20 in region-1 and other stations in the region-2. It has been found that stations in region-2 are located near the Central Mountain Range (CMR) and are associated with higher event-maximum rainfalls.

4. Random field modeling and simulation of the standardized event-maximum rainfalls

Standardized event-maximum rainfalls (SEMR) at different stations were modeled by a Pearson type III (PT3) random field with marginal density of zero expectation and unit variance. Parameters of the marginal PT3 density at individual stations were estimated using the method of L-moments. Regional parameters were then calculated as the sample-size weighted average.

Spatial covariance structure of SEMR is modeled by variogram analysis. An exponential semi-variogram model of the following form

$$\gamma(h) = 1 - \exp(-h/a) \qquad (1)$$

was used to fit the experimental variograms of SEMR of various design durations. Figure 2 illustrates the semi-variogram of the 24-hr SEMR. The parameter *a* and *h* in Eq. (1) are in unit of km. Spatial covariance (correlation) function is then represented by

$$C(h) = \rho(h) = 1 - \gamma(h) = \exp(-h/a) \quad (2)$$

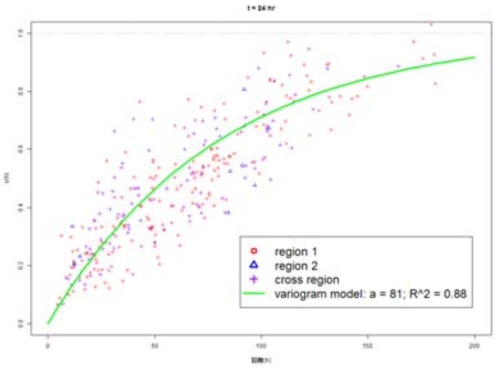

Figure 2. Semivariogram of the 24-hr SEMRs.

Stochastic simulation of the PT3 duration-specific SEMRs were conducted by transforming the PT3 random field (*X*) to a corresponding standard Gaussian random field (*Z*). The transformation requires the following conversion between the spatial covariance matrix of the PT3 random field and the covariance matrix of the standard Gaussian random field (Cheng et al., 2011):

$$\rho_{X_1X_2} \approx \left(A^2 - 6AC + 9C^2\right)\rho_{Z_1Z_2} + 2B^2\rho_{Z_1Z_2}^2 + 6C^2\rho_{Z_1Z_2}^3 \quad (3)$$

$$A = 1 + \left(\frac{\gamma}{6}\right)^2, \quad B = \left(\frac{\gamma}{6}\right) + \left(\frac{\gamma}{6}\right)^3, \quad C = \frac{1}{3}\left(\frac{\gamma}{6}\right)^2$$

where γ represents the coefficient of skewness of the PT3 random field. Stochastic simulation of standard Gaussian random field was conducted by sequential Gaussain simulation (Liou et al., 2012; Hsieh, et al., 2014). Finally, the SEMRs were calculated through the following approximation

$$X \approx z + \left(z^2 - 1\right)\left(\frac{\gamma}{6}\right) + \frac{1}{3}\left(z^3 - 6z\right)\left(\frac{\gamma}{6}\right)^2 - \left(z^2 - 1\right)\left(\frac{\gamma}{6}\right)^3 \qquad (4)$$

$$+ z\left(\frac{\gamma}{6}\right)^4 - \frac{1}{3}\left(\frac{\gamma}{6}\right)^5$$

The event-maximum rainfalls at individual stations were then obtained by statistical denormalization of SEMRs using the station-specific means and standard deviations of event-maximum rainfalls.

5. Results and discussions

For each of the eight design durations, a total of 10,000 realizations of multi-site event-maximum rainfalls were generated by the proposed stochastic simulation approach. On average, the study area experiences 2.43 typhoon events every year. Therefore, the event-maximum rainfall of *T*-year return period is associated with an exceedance probability of

$$p_E = \frac{1}{nT} \qquad (5)$$

where *n* is the average number of typhoon events per year. From the marginal density of the duration-specific SEMRs and considering the means and standard deviations of event-maximum rainfalls of individual stations, rainfall depths of *T*-year return period and given duration at individual stations can thus be determined. The results were found to be very close to the single-site frequency analysis results by a previous study, indicating good performance of the proposed approach for estimation of single-site design rainfall depths. As an example, the 24-hour rainfall depth at the Jia-sien station (1040 mm) during the typhoon Morakot is found to have a return period of 65 years, as opposed to more than 2000 years estimated by another study. A histogram of 24-hour event-maximum rainfalls at the Jia-sien station based on historical data is shown in Figure 3. The highest rainfall was produced by typhoon Morakot. However, over a 47-year period there was another higher-than 800-mm rainfall, making it unlikely for the 1040 mm rainfall depth to be associated with a higher-than 2000-year return period.

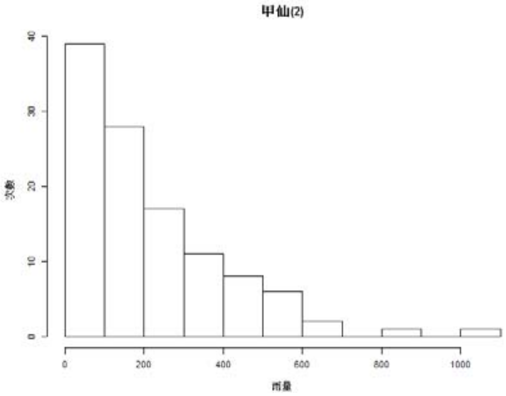

Figure 3. Histogram of the 24-hour event-maximum rainfalls at the Jia-sien station.

The proposed simulation approach takes into account the spatial correlation of rainfall extremes at different stations and thus is capable of simulating the clustering of extreme rainfalls which are often observed in real storm events. Two such simulated realizations are shown in Figure 4. The red dots represent stations having higher than 100-year rainfall depths during the events.

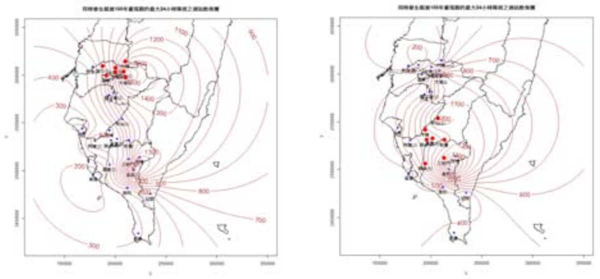

Figure 4. Examples of concurrent occurrences of 24-hour rainfall extremes at different stations simulated by the proposed approach.

A major advantage of the stochastic simulation approach presented in this study is its capability of characterizing the concurrent occurrences of rainfall extremes at different stations. Such capability enables the approach to offer an estimate of the return period for a multi-site extreme event. This is demonstrated using the 24-hour extreme rainfalls observed at several stations within the Kaoping River Basin during the typhoon Morakot. Four rainfall stations (Stations 11, 19, 21 and 25 in Fig. 1) within the Kaoping River Basin observed near or higher-than 1000 mm rainfall depths in a 24-hour period during typhoon Morakot (908, 1040, 825 and 1237 mm, respectively). Assuming a multi-site extreme event having higher-than 1000 mm rainfalls at the four stations, we found 8 out of the 10,000 simulated events met the requirements. It is equivalent to a return period of approximately 514 (≈10,000/8/2.43) years.

6. Conclusions

Concurrent occurrences of rainfall extremes at several rainfall stations are commonly observed in Taiwan. The multi-site stochastic simulation approach proposed in this study is capable of generating realizations of multi-site event-maximum rainfalls with respect to given design durations. Such realizations preserve not only the marginal densities but also the spatial correlation structure of the duration-specific event-maximum rainfalls. The 24-hour rainfall depth at the Jia-sien station during typhoon Morakot is found having a return period of 65 years. Also, the return period of the 24-hour rainfalls occurred at four rainfall stations within the Kaopig River Basin during the same event is approximately equivalent to 514 years.

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